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The Longitudinal Coupling Impedance Associated to the Pick-Up Devices for the Energy Doubler

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A longitudinal view of the pickup device which is being developed (E. Higgins)[1] for use in the Energy Doubler is shown in Fig. 1. It is made of two plates both parallel to the beam axis and each terminated to ground as shown in the figure. One of such devices is expected to be located nearby every quadrupole in the ring.

The electromagnetic analysis of a conductive plate terminated to ground (the vacuum chamber wall) was extensively done about ten years ago.[2] The major result is the following

$$Z = -8i\ M\left(\frac{\phi_o}{\pi}\right)^2\ Z_o\ P \tag{1}$$

where Z is the (complex) longitudinal coupling impedance[3], M the number of the pick-ups, ϕ_o the semi-angular aperture of a plate, Z_o is the characteristic impedance of the line formed by ground and a plate, and

$$P = \frac{\beta}{2}\ \frac{(\cos 2\phi - \cos 2\theta) + \sin 2\phi}{\cos 2\delta\phi + \cos 2\phi - 2ir\ \sin 2\phi} \tag{2}$$

where β is the ratio of the beam velocity to the light velocity c,

$$r = Z_T/Z_o = r'+ix'$$

Z_T being the termination impedance,

$$\delta = \frac{2d-\ell}{\ell}$$

d being the distance of the termination from the edge of the plate; since P is symmetric in δ we shall take

$$0 \leq \delta \leq 1$$

$\delta = 0$, termination at the center of the plate

$\delta = 1$, termination at one end of the plate.

$$\phi = \frac{\omega\ell}{2c} = \beta\theta$$

with ω the (angular) frequency induced by the beam, usually a harmonic of the revolution frequency; and ℓ is the plate length.

For the derivation and discussion of (1) and (2) we ask the reader to refer to Ref. 2. For the implication of the coupling impedance Z on the beam dynamics see Ref. 3.

The parameters for the pick-up are listed in Table I. With τ we denote the separation of a plate from ground.

We shall assume the plates are surrounded by vacuum, therefore

$$Z_o = 1/cC$$

where

$$C = \frac{2\phi_o b}{4\pi\tau}$$

is the capacitance per unit length between a plate and ground in the approximation

$$\tau << 2b, \text{ vacuum chamber diameter.}$$

Equations (1) and (2) are valid for

$$\omega \lesssim \frac{\beta c \gamma}{b} \qquad (\gamma = 1/\sqrt{1-\beta^2}).$$

A cut-off nevertheless exists for those wavelengths which are comparable or smaller than τ. For such wavelengths the plates are more or less shorted to ground ($Z_o = 0$).

The impedance of a pick-up is equivalent to a series of resonances. A resonance occurs when the denominator of (2) vanishes, i.e.

$$(\cos 2\delta\phi + \cos 2\phi) - 2x' \sin 2\phi = 0. \tag{3}$$

There are two kinds of resonances[2]:

i. Loaded Resonances

For these resonances we assume

$$\boxed{\sin 2\phi \neq 0} \quad .$$

In the limit $x' = 0$, then we have from (3)

$$\cos 2\delta\phi + \cos 2\phi = 0$$

which yields to the following resonance frequencies

$$\omega_k = \pi \frac{c}{\ell} \frac{2k-1}{1 \pm \delta} \tag{4}$$

with k a positive integer number. There are two series of resonances which correspond to the two signs at the denominator at the r.h. side of (4). Because of the cut-off we mentioned above

$$1 \leqslant k \leqslant k_{max}$$

where

$$k_{max} = \frac{\ell}{\tau}(1\pm\delta).$$

In the limit

$$\ell/4\gamma b<<1$$

at the resonance

$$P \simeq -\frac{i}{4},$$

and the shunt impedance associated to the k-th resonance is

$$Z_k = 2MZ_o\left(\frac{\phi_o}{\pi}\right)^2 \tag{5}$$

which is the same for all resonances.

One can calculate also the figure of quality of the k-th resonance as the ratio of the resonance frequency to the spread in frequency over which the real part of the impedance is induced by a factor of two. We obtain

$$\begin{aligned} Q_k &= \frac{\omega_k \ell}{4c} \quad \text{for} \quad \delta = 0 \\ &= \frac{\omega_k \ell}{2c} \quad \text{for} \quad \delta = 1 \end{aligned} \tag{6}$$

where ω_k is given by (4).

For these kinds of resonances the electric resistivity of the material which makes the plates and the walls is not important. The damping of the resonances is mainly caused by the termination impedance Z_T.

The results are summarized in Table II.

ii. <u>Unloaded Resonances</u>

These occur when simultaneously

$$\sin 2\phi = 0 \tag{7}$$

and

$$\cos 2\delta\phi + \cos 2\phi = 0 \ . \tag{8}$$

For these resonances the termination is not essential. Actually the plate appears like floating. The resonance frequencies which simultaneously satisfy (7) and (8) are

$$\omega_k = \frac{c}{\ell} \pi k \tag{9}$$

with k positive integer, but

$$k > \frac{1}{1-\delta} \ .$$

For a plate with termination in the center (δ=0), actually

$$1<k<k_{max}$$

where

$$k_{max} = 2\ell/\tau$$

which corresponds to the cutoff because of the transverse dimension τ of the line formed by the plate and the wall.

But if the termination is located next to the end of the plate in such a way that

$$\delta>1 - \frac{\tau}{2\ell}$$

then there are no resonances of this kind. <u>Therefore we suggest locating the termination at the end of the plate for the case of</u>

the pickups to be used for the Energy Doubler.

When both the conditions (7) and (8) are satisfied, the denominator at the r.h. side of (2) vanishes identically. For perfectly conductive walls and plates the shunt impedance in correspondence of the resonances (9) would be infinitely large. Damping, though, is introduced by taking into account the resistivity of the material. In proximity of the resonances (9)

$$iP = \frac{1}{4}(r + i\phi\gamma^{-2} \frac{1+\alpha\gamma^2}{\alpha\gamma^2})$$

where

$$\alpha = (1-i)\sqrt{\frac{2\pi}{\omega s}} \frac{1}{2b\phi_o Z_o} = (1-i)S$$

and s is the conductivity of the material involved.

If we take $\rho = s^{-1} = 52\mu\Omega\text{xcm}$, which is the resistivity of stainless steel at 4.2^ok, and because $\gamma \gtrsim 100$, we find that the largest contribution to the damping of the unloaded resonances is also given by the termination impedance. Then the total shunt impedance also for these resonances is given by (5). The figure of quality Q nevertheless in this case is determined by the conductivity of the material (see Ref. 2)

$$Q_k = \frac{\tau}{c} \sqrt{\frac{8\pi}{\rho} \omega_k}$$

(c.g.s. units, $\rho = 5.8\text{x}10^{-17}$ sec).

The results for the unloaded resonances are shown in Table III. These resonances are much sharper than the loaded ones.

References

1. E. Higgins, "Beam Monitoring", Doubler Status, Preliminary Draft, October 21, 1978, Fermilab

2. A.G. Ruggiero *et al.*, ISR-RF-TH/69-7, CERN, March 1969

3. A.G. Ruggiero, "Individual Bunch Longitudinal Instabilities", UPC-72, Fermilab, January 8, 1979, Unpublished

Table I. Parameters for the Pick-up Device for the Energy Doubler

ℓ	12.7 cm
τ	1 cm
ϕ_o/π	$1/\sqrt{2}$
b	4 cm
M	250
Z_o	50 ohm
Z_T	50 ohm

Table II. Loaded Resonances ($k = 1,2,...,k_{max}$)

δ	0	1
k_{max}	13	25
$\omega_k/2\pi$	1.18(2k-1) GHz	0.59(2k-1) GHz
Z_k(*)	12.5 kΩ	12.5 kΩ
Q_k	0.8(2k-1)	1.6(2k-1)

Table III. Unloaded Resonances ($k_{min} < k \lesssim k_{max}$)

δ	0	0.50	0.75
k_{min}	1	2	4
k_{max}	13	19	22
$\omega_k/2\pi$		1.18k GHz	
Z_k(*)		12.5 kΩ	
Q_k		$1.9\times10^3\ \sqrt{k}$	

(*) Total contribution from 250 devices

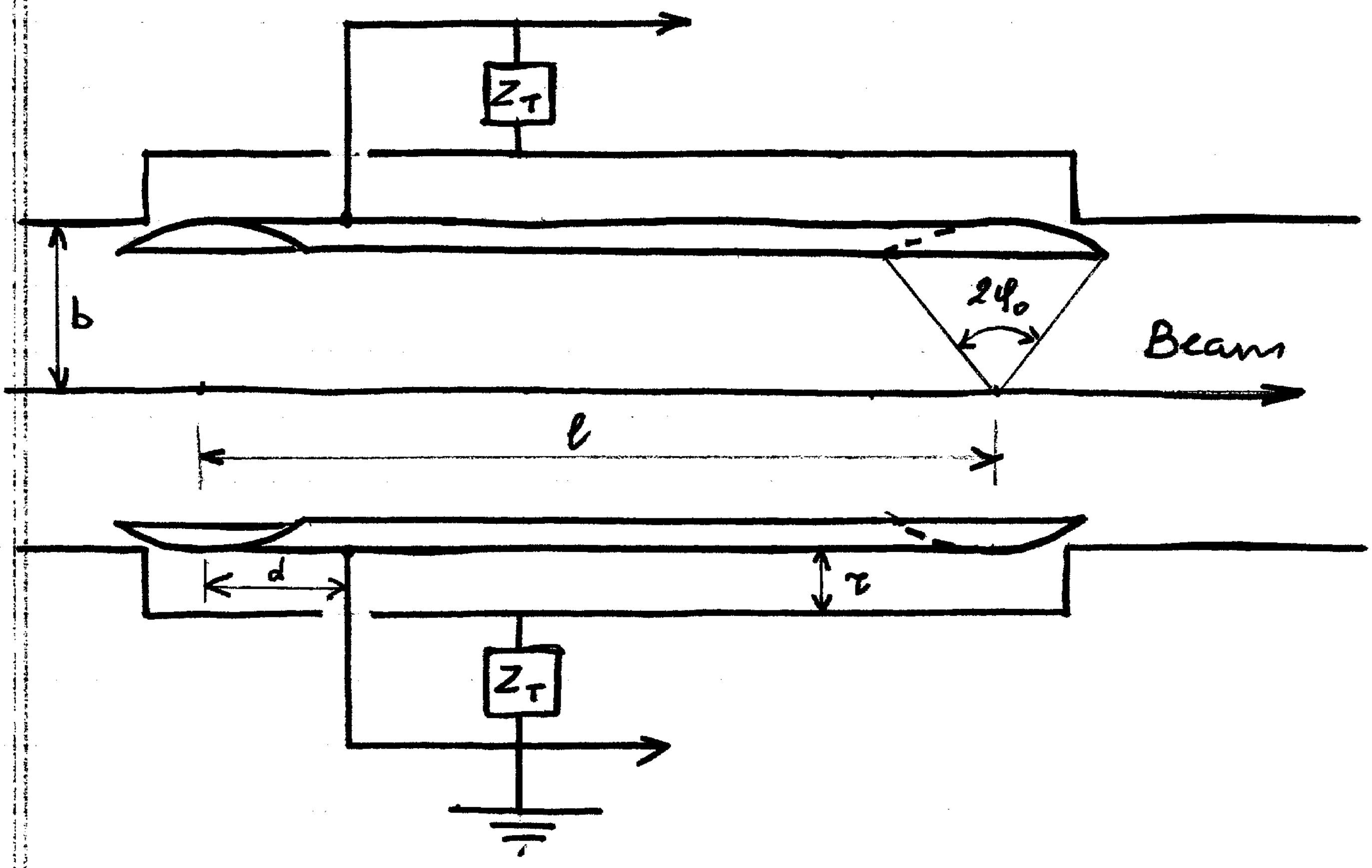

Fig. 1 Longitudinal view of a Pick-Up Device